

Amazing Amanda

Concept Lesson Guide

LESSON OVERVIEW

Overview: This is a three-part lesson, set in the context of Amanda, who claims she can predict the sum of angle measures of any polygon. In “Getting Started”, students review what they know about the sum of the measures of angles in triangles, straight lines, and around a point. In Investigation 1, students use this information to find the sum of the interior angles of convex polygons by individually drawing the polygons, subdividing them into triangles in various ways, and then finding the sum of the measures of their interior angles. In their groups, students compare their solution paths and then generalize from the various solution paths to create, and justify, algebraic equations that describe the relationship between the number of sides and the sum of the measures of the interior angles of any convex polygon. Students then verify algebraically that all of the expressions are equivalent.

In Investigation 2 students use what they discovered in Investigation 1 to find a formula for the sum of the measures of the exterior angles of any convex polygon. Investigation 3 immediately follows Investigation 2. Students are asked to find a formula for the measure of an individual interior or exterior angle of any regular polygon. They are also asked whether it is possible to find a formula for the measure of an interior and exterior angle of any irregular polygon, and if not, why not. Both Investigations 2 and 3 are discussed at the same time. In the Summary, students return to the context that opened the task – and explain Amanda’s methods.

An **Extension Task** provides an opportunity for students to expand their explorations to consider the sum of interior and exterior angles in concave polygons. The teacher notes provide possible solution paths, as well as possible errors and misconceptions that might occur when considering interior and exterior angles of concave polygons.

NCTM Standards Addressed:

- Analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Build new mathematical knowledge through problem solving.

Mathematical Goals of the Lesson:

- Develop the theorems involving angle sums of polygons.
- Develop an understanding of the connections between numeric, geometric, and algebraic patterns in geometry.
- Develop algebraic-geometric reasoning skills.
- Develop valid informal justifications and/or proofs for their mathematical generalizations.
- Write equations that describe the relationship between the number of sides and the sum of the measures of the interior and exterior angles of any convex polygon.
- Reason mathematically and use and make connections among a variety of mathematical representations.

Academic Language Goals of the Lesson:

- Develop academic vocabulary to be used in the descriptions.
- Describe the conditions in a situation algebraically, orally or in writing.
- Explain the process used in solving the task, orally or in writing.

Assumption of Prior Knowledge:

- Definition of a polygon
- Meaning of “convex” (and “concave”) polygon
- Meaning of “interior” and “exterior” angles of a polygon
- Knowledge of types of polygons and their characteristics, including regular and irregular and convex and concave polygons.
- Sum of the measures of the interior angles of a triangle = 180° .
- The sum of the measures of adjacent angles that form a straight line is 180° .
- There are 360° in a circle.

Academic Language:

- Polygon
- Convex polygon
- Measure of an angle
- Interior angle
- Exterior angle
- Regular polygon
- Irregular polygon
- Concave polygon (optional)
- Supplementary angles
- Linear pairs
- Adjacent angles
- Names of polygons:
 - Triangle
 - Quadrilateral
 - Pentagon
 - Hexagon
 - Heptagon
 - Octagon

Materials:

- Task
- Recording Sheets (provided)
- Straight edge
- Calculators

Key:

Suggested teacher questions are shown in bold print.

Questions and strategies that support ELLs are underlined and identified by an asterisk.*

Possible student responses are shown in italics.

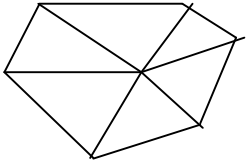
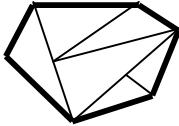

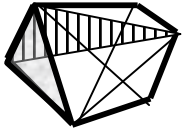
Phase	SET-UP PHASE: Setting Up the Mathematical Task
S E T U P	<p><u>INTRODUCING THE TASK</u></p> <ul style="list-style-type: none">• Ask a student to read the introduction and “Getting Started” out loud as others follow along. Give students 5 minutes to complete Getting Started in their groups.• Answer the questions as a short class discussion asking students to justify their answers with words, symbols or pictures.• The teacher should create a master class list on transparency, chalkboard, or chart paper with facts the students recall, providing diagrams to illustrate the points.• Ask a student to read Investigation 1 out loud as others follow along.• Clarify any confusions students may have, but do not suggest a method for solving the problems. <u>Ask for any words or terms that the students do not understand. Clarify the meanings so that the task will be accessible to all students.*</u>• <u>Ask several students to explain what they are being asked to do in their own words.*</u>• Specifically ask, “What is an interior angle?”, “What is a regular polygon?”, “What is an irregular polygon?” What is a “convex polygon”?• Examine the recording sheet as a class. Be sure students understand that each student will complete an individual recording sheet and that there will also be a group recording sheet that compiles the group’s work.

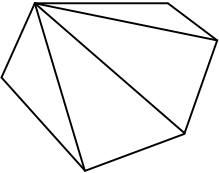
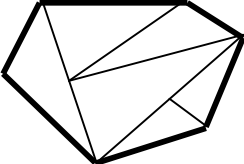
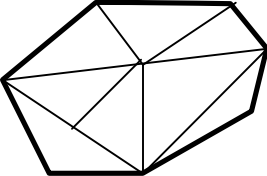
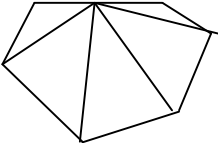
To assist ELLs’ participation in the class discussion:*

- Allow time for students to first talk in small groups (pairs) and then have the groups report to the whole class.*
- Reinforce appropriate language as students communicate their ideas (e.g. revoice a student’s contribution in mathematically and grammatically correct language). Ask students if you have captured what they said.*
- Develop a word wall and insert new terminology as it is introduced. Continually refer to it until the words become a part of student dialogue.*
- Expect students to incorporate appropriate mathematical vocabulary in their discussions once the terminology has been introduced.*

Phase	EXPLORE PHASE: Supporting Students' Exploration of the Task STRUCTURE (Investigations 1 and 2 will follow the same structure)
E X P L O R E E X P L O R E E X P L O R E	<p><u>PRIVATE THINK TIME</u></p> <ul style="list-style-type: none"> • Group members assign two or three different polygons to each student in the group and then students work individually for about 10 minutes so that they can make sense of the problem for themselves. • Each student should enter their work into their individual recording sheet – drawing the polygon, showing how they divided it into triangles, and showing their method of calculating the sum of the interior angles. • Circulate around the classroom and clarify confusions. Be careful to NOT give away too much information or suggest a way to solve the problem. <hr/> <p><u>SMALL-GROUP WORK</u></p> <ul style="list-style-type: none"> • After about 10 minutes, ask students to work with their groups to compile the polygon information onto a group-recording sheet. As a group, students should look for patterns and record their patterns under their table. Discuss in groups the answers to Questions 1 through 4. • As students are working, circulate around the room. <ul style="list-style-type: none"> ○ Be persistent in asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations. ○ Be persistent in asking students to explain their thinking and reasoning. ○ Be persistent in asking students to explain, in their own words, what other students have said. ○ Be persistent in asking students to use appropriate mathematical language. <i>Names of polygons, interior angles, exterior angles, regular or irregular, concave or convex polygons.</i> ○ Be persistent in asking students to use correct mathematical expressions for the formulas they generate to describe the relationship. <p><u>What do I do if students have difficulty getting started?</u></p> <ul style="list-style-type: none"> • Ask them to draw their assigned polygon. Ask, “How many sides does your polygon have?” “Show me the interior angles of the polygon.” “How can you divide the polygon into triangles?” “How many non-overlapping triangles are contained in your polygon?” “How can you use the information about angle sums on our chart to figure out the sum of the interior angles?” <p><u>What do I do if students finish early?</u></p> <ul style="list-style-type: none"> • Look at students’ recording sheets to determine whether they have tried both regular and irregular figures. • Challenge students to investigate whether their formula works for both concave and convex polygons. • Ask a “quiet” group member to explain the work of the group. If s/he can’t explain, challenge the group to make sure that each group member understands, leave, and return later to ask again. <hr/> <p><u>MONITORING STUDENTS’ RESPONSES</u></p> <ul style="list-style-type: none"> • As you circulate, attend to students’ mathematical thinking and to the strategies and representations used, in order to identify those responses that will be shared during the Share, Discuss, and Analyze Phase. During this phase, groups will describe how they generated the sums of both the interior and exterior angles. For this task, you will need to: <ul style="list-style-type: none"> ○ Identify students who can correctly explain and demonstrate a formula for the sum of the measurements of the interior angles of any convex polygon. Find students who used different methods, including different triangulation methods, and find students with different formulas. ○ Identify a group member or several different groups that can explain the answers to each question.

Investigation 1- Interior Angle Sum

Phase		TASK QUESTIONS 1 - 4						
		<p>As a group, combine your results on a single recording sheet and answer these questions:</p> <ol style="list-style-type: none"> 1. How did group members subdivide their polygons into triangles? Did everyone do it in the same way? If different, how did that affect your calculations? 2. Does whether the polygon is regular or irregular affect the sum of the angle measures? Why or why not? 3. What patterns do you notice as you explored the problem? 4. What is the relationship between the number of sides of the polygon and the sum of the measures of the interior angles? Express this relationship algebraically and explain how you know that your expression will work for ANY convex polygon. 						
		Possible Solutions	Possible Questions	Misconceptions/Errors	Questions to Address Misconceptions/Errors			
E X P L O R E E X P L O R E E X P L O R E	<p>Look for indicators of students' understanding of:</p> <ul style="list-style-type: none"> • Correctly drawing their assigned polygons and partitioning them into triangles. • Correctly counting the non-overlapping triangles. • Correctly identifying the interior angles of their polygons and identifying the angles in the triangles that do – and those that do not – make up the interior angles of the polygon. <p><u>Drawing line segments to the vertices from a common interior point</u></p>  <p style="text-align: center;">Method 1</p> <p>$S = 6 \times 180 - 360 = 180n - 360$ <i>I can always draw the same number of triangles as the number of sides and each of them has 180°. But then I have to subtract 360° because I included the angles in the middle.</i></p>		<p>Ask questions such as:</p> <ul style="list-style-type: none"> • How did you divide your polygon into triangles? How many triangles did you make? • How did you find the sum of the interior angles of the polygon? • How could you use your technique to find the sum of the interior angles for any convex polygon? • How do you know your method will always work? 		<ul style="list-style-type: none"> • Not realizing that some of the angles in their triangles are not included in the interior angles of the polygon.  <p>That is, $S = 6 \times 180$ (the correct calculation using this diagram would be $S = 6 \times 180 - 2(180)$ for the “extra” angles).</p> <ul style="list-style-type: none"> • Partitioning the polygons into non-overlapping shapes, but not all of the interior shapes are triangles.  <p>That is, $S = 10 \times 180 - \text{????}$</p> <ul style="list-style-type: none"> • Partitioning the polygons into overlapping triangles (i.e., including the same angle in several triangles).  <p>$\angle A$</p> <p>That is, $S = 8 \times 180$</p>		<ul style="list-style-type: none"> • (for all) How many triangles did you make? Show me each of them. • How did you calculate the sum of the interior angles? • Which of the angles are part of the “interior angles” of the polygon? what can we do so we only include the IAs? • How did you calculate your sum? • Which angles are included in this sum? Are they all IAs? • How can you adjust your sum (they will see that they will not be able to compensate for all of the “extra” angles)? • (point to one interior angle, e.g., $\angle A$) Which of your triangles contains this angle? Show me the part of each triangle that you are including. What do you notice? 	

Phase		TASK QUESTIONS 5 - 8				
		<p>As a group, combine your results on a single recording sheet and answer these questions:</p> <ol style="list-style-type: none"> 1. How did group members subdivide their polygons into triangles? Did everyone do it in the same way? If different, how did that affect your calculations? 2. Does whether the polygon is regular or irregular affect the sum of the angle measures? Why or why not? 3. What patterns do you notice as you explored the problem? 4. What is the relationship between the number of sides of the polygon and the sum of the measures of the interior angles? Express this relationship algebraically and explain how you know that your expression will work for ANY convex polygon. 				
		Possible Solutions	Possible Questions	Misconceptions/Errors	Questions to Address Misconceptions/Errors	
E X P L O R E	<p><u>Drawing diagonals to non-adjacent vertices from a single vertex</u></p>  <p>Method 2</p> <p>$S = 4 \times 180 = 180(n - 2)$ <i>I can always draw two less triangles than the number of sides and each of them has 180°</i></p>		<p>Ask questions such as:</p> <ul style="list-style-type: none"> • How did you divide your polygon into triangles? How many triangles did you make? • How did you find the sum of the interior angles of the polygon? • How could you use your technique to find the sum of the interior angles for any convex polygon? 	<p><u>These are not misconceptions or errors, however their non-systematic nature will make it difficult for students to generalize their results.</u></p> <ul style="list-style-type: none"> • Not using a systematic procedure to partition their polygon that could be readily generalized to n-gons  <p>$S = 6(180) - 180 - 180$</p>  <p>$S = 8(180) - 360 - 180$</p>	<ul style="list-style-type: none"> ▪ How did you decide how to divide your polygon into triangles? ▪ Could you figure out ahead of time how many triangles you will create for any size polygon? ▪ Can you think of a systematic method that you could use to divide any polygon into triangles? (If they do not come up with one, they will see other methods as they work in their group.) 	
	E X P L O R E	<p><u>Drawing diagonals to non-adjacent vertices from a common point on one side</u></p>  <p>Method 3</p> <p>$S = 5 \times 180 - 180 = 180(n - 1) - 180$ <i>I can always draw one less triangle than the number of sides and each of them has 180°. I have to subtract another 180° because I've too many angles (the ones around the point).</i></p>				
	E X P L O R E					

Share, Discuss, and Analyze — Investigation 1

Orchestrating the mathematical discussion: a possible Sequence for sharing student work, Rationale and Mathematical Ideas to achieve the goals of the lesson, and possible Student Responses that demonstrate understanding.

Phase	Sequencing of Student Work	Rationale and Mathematical Ideas	Possible Questions and Student Responses
S H A R E D I S C U S S A N A L Y Z E	<p>Each group (or select groups) describes ONE strategy for determining the sum of the interior angles and links their strategy to their triangulation method.</p> <ol style="list-style-type: none"> 1. A group should be chosen that used triangulation from one vertex. 2. Next, a group should be chosen that used triangulation from a central point. 3. Next a group should be chosen that used triangulation from a point on one side. 4. Any other non-standard methods that yielded the correct sum should also be explained. 	<p>This discussion will lay the foundation for the <i>sum of the interior angles of a polygon theorem</i>.</p> <p>The relationships between the diagrams and the algebra should be discussed and demonstrated reinforcing geometric-algebraic reasoning.</p>	<p>How did you partition your polygon to determine the sum of the interior angles?</p> <ul style="list-style-type: none"> • <i>I used diagonals from one vertex to partition the polygon into triangles and used the fact that triangles have 180° to determine the sum.</i> • <i>I made triangles from a central point with each side of my polygon as the base of each triangle but I had to subtract 360° from my sum.</i> <p>Why did you subtract 360°?</p> <p><i>Because the angles around the central point are not interior angles of the polygon. And, since these angles form a circle, I knew that the sum of their measures was 360°.</i></p> <ul style="list-style-type: none"> • <i>I made triangles by drawing lines from a point on one side to all of the vertices. I multiplied the number of triangles by 180° but also had to subtract 180° to get my sum.</i> <p>Why did you subtract 180°?</p> <p><i>Because the angles around the point on the side are not interior angles of the polygon. And, since these angles form a straight line, I knew that the sum of their measures was 180°.</i></p> <ul style="list-style-type: none"> • <i>I made triangles by drawing lines until all of the non-triangular shapes were divided up. I multiplied the number of triangles by 180 but then I had to subtract all the angles that weren't part of the interior angles of the polygon.</i> <p>What patterns did you notice?</p> <ul style="list-style-type: none"> • <i>I always had 2 less triangles than the number of sides</i> • <i>I had the same number of triangles as sides but I had to subtract 360 which is like subtracting two triangles</i> • <i>I had one less triangle than the number of sides but I had to subtract 180 which is like subtracting another triangle.</i> • <i>It was hard to find patterns because I divided each polygon up differently. I see that the other methods were better (i.e., more systematic, easier to generalize).</i>

Phase	Sequencing of Student Work	Rationale and Mathematical Ideas	Possible Questions and Student Responses
S H A R E D I S C U S A N A L Y Z E	<p>5. If the group did not write a general formula for their method, the whole class should be pressed to write a general formula that reflects how each triangulation method would be used to calculate the sum of interior angles for an n-gon.</p> <p>6. The whole class should be asked to verify that the algebraic expressions for the sum of the interior angles to determine equivalency.</p> <p>Answers to Questions 1, and 2 can be discussed as a class if the group presentations did not adequately cover the mathematics.</p>	<p>This discussion will lay the foundation for the <i>sum of the interior angles of a polygon theorem</i>.</p> <p>Students should be able to verify the equivalence of the different formulas found by algebraically simplifying each expression to yield $S = 180n - 360$, or by showing that they are each equivalent to $S = 180(n - 2)$ since $180n - 360 = 180n - 180(2) = 180(n - 2)$.</p>	<p>How did you (or can we) generate a formula using your method?</p> <ul style="list-style-type: none"> • <i>I used a variable for the number of sides and subtracted 2 since I had two less triangles, then multiplied by 180.</i> • $S = 180(n - 2)$ • <i>I used a variable for the number of sides then multiplied by 180 and subtracted 360.</i> • $S = 180n - 360$ • <i>I used a variable for the number of sides and subtracted 1 since I had one less triangle. Then I multiplied by 180 and subtracted 180.</i> $S = 180(n - 1) - 180$ <p>Are these three formulas equivalent? Why or why not?</p> <ul style="list-style-type: none"> • <i>$S = 180n - 360$, $S = 180(n - 1) - 180$ and $S = 180(n - 2)$ are equivalent if you distribute the 180 across the parentheses and combine like terms; they each become $180n - 360$.</i> • <i>You can also think of 360 as $180(2)$ so you get $S = 180n - 360 = 180n - 180(2) = 180(n - 2)$.</i> • <i>You can do the same with the other expression since 180 equals $180(1)$.</i> • <i>Subtracting 360 is like subtracting two triangle totals from the sum and subtracting 180 is like subtracting one triangle total from the sum – so that’s like $n - 2$ triangles.</i> <p>Why was it easier to find formulas using the systematic methods of dividing up the polygons?</p> <ul style="list-style-type: none"> • <i>Because you could describe how you would divide up any polygon, and you could figure out how many triangles you would make using each method if you knew the number of sides.</i> <p>Does it matter whether the polygons are regular or irregular?</p> <ul style="list-style-type: none"> • <i>No, they all worked the same.</i>

Investigation 2: Exterior Angle Sum

Phase	TASK QUESTIONS 1 - 3			
<p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p>	Possible Solutions	Possible Questions	Misconceptions/Errors	Questions to Address Misconceptions/Errors
	<p>Look for indicators of students' understanding:</p> <ul style="list-style-type: none"> • Students draw the exterior angles correctly, with only one at each vertex. • Students realize that each exterior angle makes a linear pair (180°) with (i.e., is the supplement of) each interior angle. • Students realize that they can use what they learned in Investigation 1, as well as the relationship between the measure of an interior and its exterior angle, to determine the sum of the exterior angles. • Students' calculation methods yield 360° for each polygon (see teacher notes on page 15). 	<p>Ask questions such as:</p> <ul style="list-style-type: none"> • Describe how you drew the exterior angles. • Which of these angles are exterior angles? • What is the relationship between the measures of the interior and exterior angles at each vertex? • How did you calculate the sum of the exterior angles? How did you use what you learned in Investigation 1 to help you think about this situation? • Explain in your own words what ____ (another student) said.* 	<ul style="list-style-type: none"> • Using two exterior angles at each vertex. • Not drawing the exterior angles correctly (some students draw the vertical angle match to the interior angle). • Not being able to represent the situation algebraically. 	<ul style="list-style-type: none"> • Have you used just one exterior angle at each vertex? • Is your exterior angle supplementary to the interior angle? • How many pairs of supplementary angles do you have in your polygon? • Can you write a number sentence for your specific polygon? • Can you write an algebraic sentence for any convex polygon?

Investigation 3: Regular Polygons

Phase	TASK QUESTIONS 1 & 2			
	<p>Can you predict the measure of a single interior or exterior angle of any regular polygon?</p> <ol style="list-style-type: none"> Using what you discovered in Investigations 1 and 2, is it possible to find a formula for the measure of a single interior or exterior angle of any regular polygon? If yes, what is the formula? If no, why is it not possible? Is it possible to find a formula for the measure of individual interior or exterior angles of an irregular polygon? Why or why not? 			
E X P L O R E E X P L O R E E X P L O R E	Possible Solutions	Possible Questions	Misconceptions/ Errors	Questions to Address Misconceptions/Errors
	<p>Look for indicators of students' understanding:</p> <ul style="list-style-type: none"> Students use their work from Investigations 1 and 2 to determine the angle measures of regular polygons. Students are able to write a formula for the interior and exterior angles of regular polygons. <p>Interior angle:</p> $180(N-2) / N$ <p>Exterior angle:</p> $360 / N$ <ul style="list-style-type: none"> Students realize that this is not possible with an irregular polygon. 	<p>Ask questions such as:</p> <ul style="list-style-type: none"> What is a regular polygon? An irregular polygon? Can you think of a regular or equilateral triangle first? What is the total interior angle measure? What is the measure of each angle? What is the total exterior angle measure? What would one exterior angle measure? Are the two angles supplements? Why is that important? Use each of the regular polygons on your recording sheet — can you determine the interior angle measure for one of the angles? the exterior angle measure for one of the angles? Explain in your own words what ____ (another student) said.* 	<ul style="list-style-type: none"> Thinking that you can predict the interior and exterior angles for irregular polygons. Not being able to come up with a general formula. 	<ul style="list-style-type: none"> In an irregular polygon can you always determine each angle measure? Can you write a number sentence for your specific regular polygon? Can you write an algebraic sentence for any regular polygon?

Share, Discuss, and Analyze – Investigations 2 and 3

Orchestrating the mathematical discussion: a possible Sequence for sharing student work, Rationale and Mathematical Ideas to achieve the goals of the lesson, and possible Student Responses that demonstrate understanding.

Phase	Sequencing of Student Work	Rationale and Mathematical Ideas	Possible Questions and Student Responses
S H A R E D I S C U S S A N A L Y Z E	<p>Investigation 2: Each group (or select groups) describes ONE strategy for determining the sum of the exterior angles of any convex polygon.</p> <ol style="list-style-type: none"> 1) Different strategies should be shown to the class. 2) Groups should explain the patterns they noticed and compare their algebraic expressions for the sum of the exterior angles to determine equivalency. <p>Investigation 3: Groups with different explanations should explain their thinking and demonstrate their formulas and how they derived them, using their work on Investigation 1 and 2, for the individual interior and exterior angles of regular (and irregular) polygons.</p>	<p>This discussion will lay the foundation for the <i>sum of the exterior angles of a polygon theorem</i>.</p> <p>Students should be able to explain the equivalency of the different formulas found by the different groups.</p> <p>The relationships between the diagrams and the algebra should be discussed and demonstrated reinforcing geometric-algebraic reasoning.</p> <p>Regular and irregular polygons are compared and contrasted.</p>	<p>How did you determine the sum of the exterior angles?</p> <ul style="list-style-type: none"> • <i>I used the fact that each exterior angle makes a linear pair with one interior angle, so they are supplementary and add to 180°. So, I multiplied 180 by the number of sides, then subtracted the measure of the interior angles.</i> • <i>I wrote each exterior angle as the supplement of the interior angle and then wrote the sum. I then regrouped the terms and simplified. I knew also that the sum of the interior angles was $180n - 360$.</i> <p>What patterns did you notice?</p> <ul style="list-style-type: none"> • <i>I always got 360°.</i> <p>How did you generate a formula?</p> <ul style="list-style-type: none"> • <i>I used a variable for the number of vertices multiplied by 180 and then I subtracted the interior angles from that product. $180n - (n-2) \times 180 = 2 \times 180 = 360$.</i> • <i>I knew that when I simplified my expression I would get $180n$ and I then had to take away the sum of the interior angles which was $180n - 360$. $180n - (180n - 360) = 180n - 180n + 360 = 360$.</i> <p>Does it matter whether the polygons are regular or irregular?</p> <ul style="list-style-type: none"> • <i>No, they both worked the same.</i> <p>Could you find the individual angle measures for the interior and exterior angles of regular and irregular polygons? Explain.</p> <ul style="list-style-type: none"> • <i>I could find the individual interior and exterior angle measures of regular polygons because by definition all the angles are equivalent in regular polygons. So their supplements will be equivalent too.</i> • <i>In irregular polygons the angles are not all equivalent so it is impossible to determine individual measures.</i> <p>What formulas did you use to determine the individual angles?</p> <ul style="list-style-type: none"> • <i>I used $180(n - 2) / n$ for interior angles</i> • <i>I used $360/n$ for the exterior angles where n is the number of sides or angles of the polygon.</i>

Share, Discuss, and Analyze – Task Summary

Using their recording sheets, students should first discuss and answer the two questions in their group before the final teacher led discussion.

Orchestrating the mathematical discussion: a possible Sequence for sharing student work, Rationale and Mathematical Ideas to achieve the goals of the lesson, and possible Student Responses that demonstrate understanding.

Revisiting the Mathematical Goals of the Lesson:

- Develop the theorems involving angle sums of polygons.
- Develop an understanding of the connections between numeric, geometric, and algebraic patterns in geometry.
- Develop algebraic-geometric reasoning skills.
- Write equations that describe the relationship between the number of sides and the sum of the measures of the interior and exterior angles of any convex polygon.
- Reason mathematically and use and make connections among a variety of mathematical representations.

Based on Investigations 1, 2 and 3:

- 1) How could Amanda predict the sum of measures of the interior angles of a polygon if she only knew the number of sides?
- 2) How could Amanda predict the sum of the exterior angles of a convex polygon if she only knew the number of sides?
- 3) What additional relationships exist for regular convex polygons?

If any students in the class have explored concave polygons as an extension to Investigation 1 and/or Investigation 2 they should explain their work to the class after the summary has been completed. Explanations should include diagrams and symbols. The teacher may also lead a class exploration of concave polygons with the class if no students explored their interior angle sums or exterior angle sums as an extension.

Phase	Sequencing of Student Work	Rationale and Mathematical Ideas	Possible Questions and Student Responses
<p style="text-align: center;">S H A R E</p> <p style="text-align: center;">D I S C U S S</p> <p style="text-align: center;">A N A L Y Z E</p>	<p>This is a teacher-led discussion of the whole task. Students should be asked to recap their work on the task and to summarize their thinking.</p>	<p>Students should explain their thinking to each question using their recording sheets and mathematical reasoning to support their answers.</p> <p>Specific connections should be made between the geometric diagrams, the measures, the sums and the algebraic formulas.</p> <p>There should also be an encore of the discussion of the different formulas and their equivalencies.</p>	<p>This phase is the debriefing of the entire task. The “Key Questions” are the framing questions for the class discussion.</p> <p>Q1: How could Amanda predict the sum of measures of the interior angles of a polygon if she only knew the number of sides?</p> <p>She can predict the measure of the sum of the interior angles. The formula is $180(n - 2)$ where n is the number of sides of the polygon.</p> <p>Q2: How could Amanda predict the sum of the exterior angles of a convex polygon if she only knew the number of sides?</p> <p>She can predict that the sum of the exterior angles will be 360° regardless of the number of sides.</p> <p>Q3: What additional relationships exist for regular convex polygons?</p> <p>If a polygon is regular she can divide the sum of the interior angles of the polygon by n, the number of angles and predict the interior angle measure.</p> <p>$180(n-2) / n$</p> <p>If a polygon is regular you can divide 360 by n and determine the exterior angle measure.</p> <p>$360/n$</p>

Extension Question

What relationships exist between angle measures of CONCAVE polygons and the number of sides?

CLOSURE

Have students reflect on the mathematics of the lesson; find links to math that they have explored before, i.e., polygons, supplemental angles, interior angles, exterior angles, regular and irregular figures.

It is important for students to step back and reflect on the ideas that surfaced and to situate their learning within past experiences, and to think forward to ways that they might build on these ideas in future tasks. This helps them to focus on the interconnectedness of mathematical ideas.

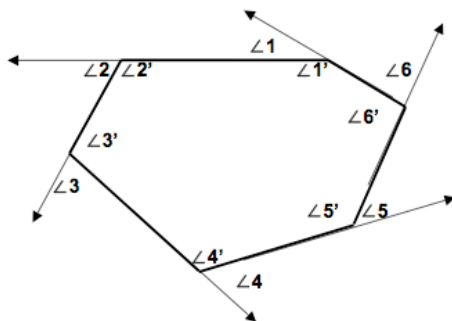
HOMEWORK

- Which regular polygons tessellate the plane and why? Why don't the others?

Teacher Notes: Exterior Angles of Convex Polygons

An exterior angle of a polygon is formed by extending one side of the polygon forming a linear pair (supplementary angles) with one of the interior angles. While it is possible to draw TWO (equal) exterior angles at each vertex of a polygon, the sum of the exterior angles uses only ONE exterior angle at each. The convention is to draw all exterior angles pointing in either the clockwise or counterclockwise direction.

Since each exterior angle is the supplement of the corresponding interior angle, students can use the expression they found for the sum of the interior angle measures to find the sum of the exterior angles:



Method 1:

Represent the sum of the measures of the linear pairs which includes both the interior and exterior angles and then subtract the sum of the interior angles, which was found in Investigation 1.

$$S = 180n - 180(n-2) = 180n - 180n + 360 = 360^\circ$$

Method 2:

The sum of exterior angles can be determined by representing each exterior angle as the supplement of its paired interior angle and then simplifying:

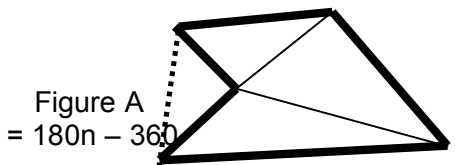
$$S = (180 - m\angle 1') + (180 - m\angle 2') + (180 - m\angle 3') + (180 - m\angle 4') + (180 - m\angle 5') + (180 - m\angle 6') = 6(180) - (m\angle 1' + m\angle 2' + m\angle 3' + m\angle 4' + m\angle 5' + m\angle 6') \rightarrow n(180) - 180(n-2) = 2(180) = 360^\circ.$$

Additional Teacher Notes: Concave polygons

Teachers have a choice, depending on their class, to have students investigate just convex polygons or both convex and concave polygons in either Investigation 1 and/or 2. A polygon is **concave** if a diagonal can be drawn that contains points in the exterior of the polygon (see the dashed diagonal in Figure A). In Investigation 1 triangulating either convex or concave polygons to investigate the sum of the interior angles of any polygon will yield the equivalent formulas. However, students should be challenged to show that all of the interior angles of a concave polygon are included in the triangulation process. Figures A, B and C account for all interior angles. Figure C does not; Figure C includes all of the interior angles, but also includes a linear pair which will need to be subtracted.

Concave Pentagon

$S = 3 \times 180 = 540^\circ$
 $= 180(n - 2)$



$S = 5 \times 180 - 360 = 540^\circ$

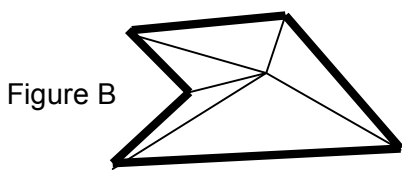
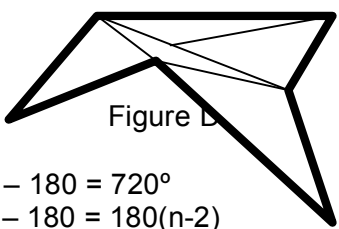
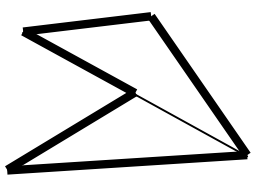


Figure C

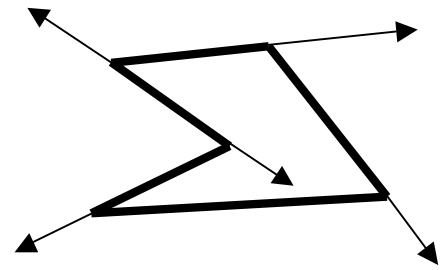


$S = 5(180) - 180 = 720^\circ$
 $= 180(n-1) - 180 = 180(n-2)$



In contrast, in Investigation 2, drawing the exterior angles of a concave polygon can be tricky since the exterior angle may in fact lie in the interior of the polygon and in that case the angle measure is negative (the difficulty in this context is seeing, and understanding, that the sum of the interior and exterior angles at a given vertex is always 180°). A convincing illustration of this point can be seen at <http://www.mathopenref.com/polygonanglerelation.html>. Numerically, if you add the exterior angles, they still add to 360, you just have to remember to add the negative angles correctly. Algebraically, the formula $180n - 180(n-2)$ still holds once the student uses the correct measures of all of the exterior angles. See also Dr. Math <http://mathforum.org/library/drmath/view/62228.html>.

Concave Pentagon With 5 Exterior Angles Drawn



Thus, teachers may choose to reserve work with concave polygons for extension activities in Investigation 2 or in both Investigations 1 and 2, or assign them to students who finish early and then have those students demonstrate their work on concave polygon sums to the class. (This option may be dependent upon the curriculum or standards in place).

REFLECTION QUESTION(S):
EXTENSIONS: