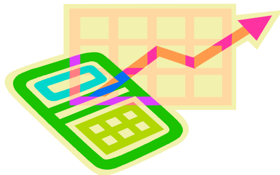

SHAPES OF QUADRATICS

In the following lessons, you will investigate how you can use the graph of the basic quadratic function $y = x^2$ to describe the graphs of more complex quadratic functions.

GETTING STARTED: Revisit the graph of $y = x^2$

Review the characteristics of the graph of the basic quadratic function:

- Enter $y = x^2$ into your graphing calculator. View the graph. Recall that this graph is called a parabola.
- What do you notice about the graph? Record your observations and discuss them with the members of your group.

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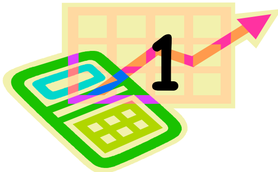

INVESTIGATING THE SHAPE OF QUADRATIC FUNCTIONS FROM EQUATIONS TO GRAPHS

INVESTIGATION 1: FROM EQUATIONS TO GRAPHS I

For graphs of each of the functions below, you will explore the appearance of the graph and its location on the coordinate plane, and how they are related to the graph of the basic function $y = x^2$.

- $y = ax^2$
- $y = x^2 + c$
- $y = ax^2 + c$

For ***each*** of the three equations:

Use your graphing calculator to graph different functions of each of the three forms. Substitute different values for *a* and/or *c*. Use a variety of values including ones that are greater than 1, between 0 and 1, positive and negative. Record your observations on the Investigation 1 Recording Sheet.

Divide the workload among members of your group. Graph a set of equations on the same screen by entering them in the calculator's "Y=" list. Keep $y = x^2$ as the first equation in the list. Use the standard viewing window.

As a group, use your graphs to answer these questions:

a) How are the graphs similar to, and different from, the graph of the basic function $y = x^2$?

b) How are these graphs similar to, and different from, each other?

c) How do changes in the values of *a* and *c* affect:

- the appearance of the graph and
- the location of the vertex of the graph in the coordinate plane?

Check your conclusions. Create another function of the same form and predict what the graph will look like before graphing it on your calculator.

Be prepared to discuss your results with the class.

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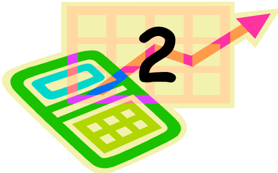

INVESTIGATION 2: FROM EQUATIONS TO GRAPHS II

Use the same process as in Investigation 1 to explore these forms of quadratic equations. Again, you will investigate how these graphs are related to the graph of the basic function $y = x^2$.

- $y = (x + h)^2$
- $y = a(x + h)^2$
- $y = a(x + h)^2 + k$

For ***each*** of the three forms:

Substitute different values for *a*, *h* and/or *k*. Use a variety of values including ones that are greater than 1, between 0 and 1, between 0 and -1, and less than -1. Record your observations on the Investigation 2 Recording Sheet.

Divide the workload among members of your group. Graph a set of equations on the same screen by entering them in the calculator's "Y=" list. Keep $y = x^2$ as the first equation in the list. Use the standard viewing window.

As a group, use your graphs to answer these questions:

a) How are the graphs similar to, and different from, the graph of the basic function $y = x^2$?

b) How are these graphs similar to, and different from, each other?

c) How do changes in the values of a, h and/or k affect:

   – the appearance of the graph and

   – the location of the vertex of the graph in the coordinate plane?

d) How are these graphs similar to, and different from, the graphs of the equations in Investigation 1? Specifically, how are the effects of the values of *a* and *c* in the equations in Investigation 1 similar to, or different from, the effects of *a*, *h*, and *k* in the equations above?

Check your conclusions. Create another function of the same form and predict what the graph will look like before graphing it on your calculator.

Be prepared to discuss your results with the class.

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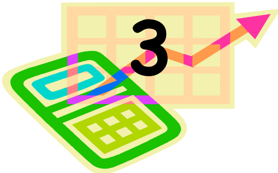

NAME THAT GRAPH

INVESTIGATION 3: FROM GRAPHS TO EQUATIONS

Objective: To write an equation that describes a mystery graph.

Grouping: 2 players

Materials: Graphing calculator for each player, ***NAME THAT GRAPH*** recording sheets

Directions: Player 1 creates a mystery graph by entering a quadratic equation into his or her calculator. (Be sure to clear the Y= list before each round.) Equations may be of any of the six forms you explored. Coefficients and constants are selected from the following list:

-10, -5, -4, -3, -2, -1, -½, -¼, - 0.1, 0, 0.1, ¼, ½, 1, 2, 3, 4, 5, 10

Player 1 displays the ***graph*** of the quadratic function (NOT the actual equation) in the standard viewing window, or give the other player the correct window values if you changed the viewing window. This is the mystery graph.

Player 2 then tries to find the equation that produced the mystery graph.

- Player 2 predicts the equation on the recording sheet.
- Player 1 graphs the predicted equation on his or her calculator and displays it along with the mystery graph. (Player 2 also may graph the predicted equation on his or her own calculator to check that Player 1 entered the equation correctly. Be sure to use the same viewing window on both calculators.)
- Player 2 compares the graph of the predicted equation to the mystery graph. (Player 2 may ask Player 1 to change the viewing window to better compare the graphs.) If the graphs don't match, Player 2 revises his or her equation to get a better fit. Player 1 then displays the graph of the new predicted equation along with the mystery graph.

Player 2 continues to revise the predicted equation and compare graphs until he or she believes that the predicted equation is the equation of the mystery graph. At that point, Player 1 reveals the equation of the mystery graph. Player 2 receives 5 points if the final equation is correct, and loses 1 point for each incorrect prediction equation. Players then reverse roles: Player 1 tries to find the equation of the mystery graph that Player 2 creates.

Play continues until each player has tried to find equations for 3 mystery graphs, or until time runs out.

The winner is the player with the highest total score.

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Analyzing Your Strategies:

How did you revise your predicted equation if the graph:

1) Opened in the wrong direction?
2) Did not have the correct position above or below the x-axis (i.e., vertical translation)?
3) Did not have the correct position to the right or left of the y-axis (i.e., horizontal translation)?
4) Was narrower or wider than the mystery graph?

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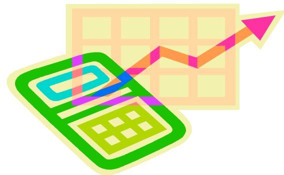

WRAP-UP:
Pulling it All Together

Think about each of these six forms of the quadratic equation and the basic shapes and locations of their graphs:

- $y = ax^2$
- $y = x^2 + c$
- $y = ax^2 + c$
- $y = (x + h)^2$
- $y = a(x + h)^2$
- $y = a(x + h)^2 + k$

1) How does the value of *a* affect the appearance and location of the vertex of the graph?
2) How does the value of the constant term *c* or *k* affect the shape and location of the vertex of the graph?
3) How does varying *h* in $a(x + h)^2 + k$ affect the location of the vertex of the graph? How do the effects of *a, h, and k* in $a(x + h)^2 + k$ How do the graphs of these functions compare to the graphs of $y = ax^2$, $y = x^2 + c$ and $y = ax^2 + c$?
4) Can a quadratic function have ***both*** a maximum and minimum point? Why or why not?
5) How can you tell from the equation whether a quadratic function has a maximum or minimum point on the graph?
6) Does the graph of every quadratic function have a y-intercept? How can you use the equation to determine the y-intercept?
7) Consider the equation: $y = x^2 - 4x + 7$. Todd says, "The graph has a minimum point of (0, 7) because c = 7." Lynn says, "I don't think so. *C* is equal to 7, but there's an "x" term in the equation. I think that might make a difference." Who is correct? Use your graphing calculator to investigate.

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