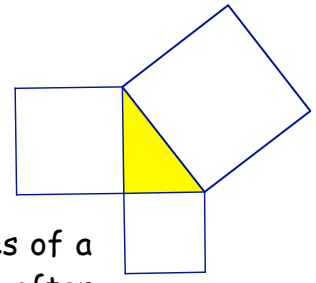
Squaring Triangles



The diagram at the right is often used to illustrate a famous mathematical relationship between the areas of the squares on the sides of a right triangle—the Pythagorean theorem. Although it is named after Pythagoras, a Greek mathematician who lived in the 6th century B.C., there is evidence that other ancient cultures, including the Chinese, Egyptians, and Babylonians, also knew about and used this relationship. There are more than 300 different proofs of this theorem. In this lesson, you will examine two of the best-known proofs.

Getting Started: Review what you know about the Pythagorean theorem

- Individually, write down everything you know about the Pythagorean theorem.

- In your groups, create a combined group list:
  - Select one person to be the recorder.
  - Group members take turns sharing one thing they wrote down.
  - The recorder writes down only the statements that all the group members agree are related to the Pythagorean theorem.
  - Continue until all group members' ideas have been discussed.

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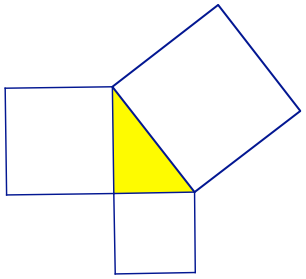
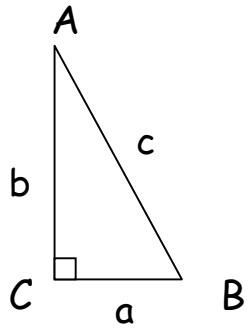


Investigation 1: Proving the Pythagorean Theorem: There's More than One Way to Square a Triangle!

Figures 1 and 2 below were used to construct one of the best-known visual proofs of the Pythagorean theorem. **How can it be done?**

Consider any right triangle ABC with legs of lengths a and b, and hypotenuse of length c. **Given:** all right triangles in Figures 1 and Figure 2 are congruent with right triangle ABC. The outside borders of Figures 1 and 2 are straight lines.



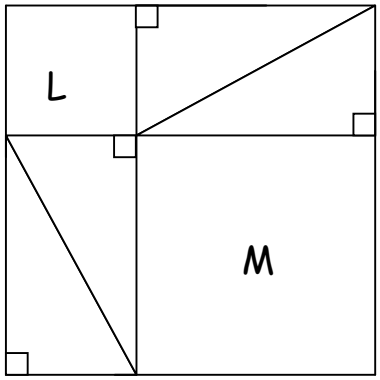


Figure 1

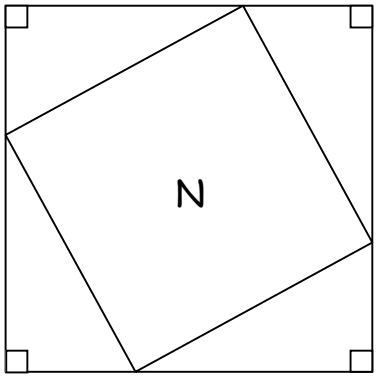


Figure 2

1. **Working with your group:** Using the information that you have been given above, what can you conclude about the shapes and areas in Figures 1 and 2? Try to provide a justification for each of your conclusions.

Conclusions:	**Justifications:**

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2. Key Question: Use what you have established about Figures 1 and 2 to construct a proof of the Pythagorean Theorem. You will need to justify all of your statements, not just assume them. Hint: both visual proofs and algebraic proofs are possible.

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Extension:

3. The Hindu mathematician Bhaskara (1114-1185) developed several proofs of the Pythagorean theorem. He developed one from Figure 2 alone. Use what you established about the shapes in Figure 2 to construct a proof of the Theorem.

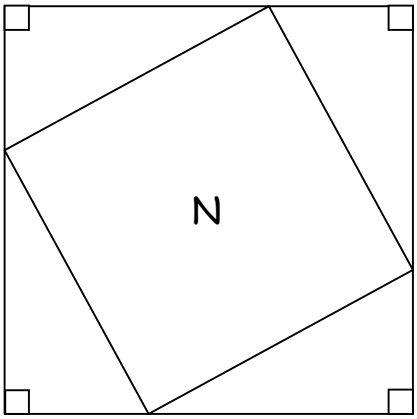

Figure 2

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Investigation 2: A Chinese Proof

An ancient Chinese text contains this diagram, which is constructed of 4 congruent right triangles:

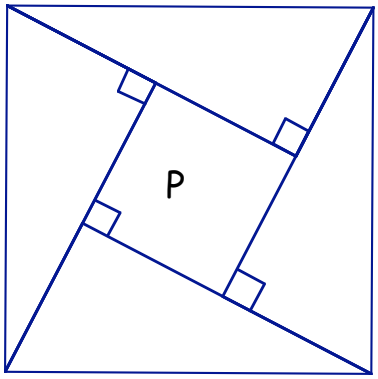

Figure 3

1) What can you conclude about the large quadrilateral in Figure 3 and the shapes in it? Remember, you will need to prove your conclusions, and not just state them.

2) How can this figure be used to prove the Pythagorean Theorem?

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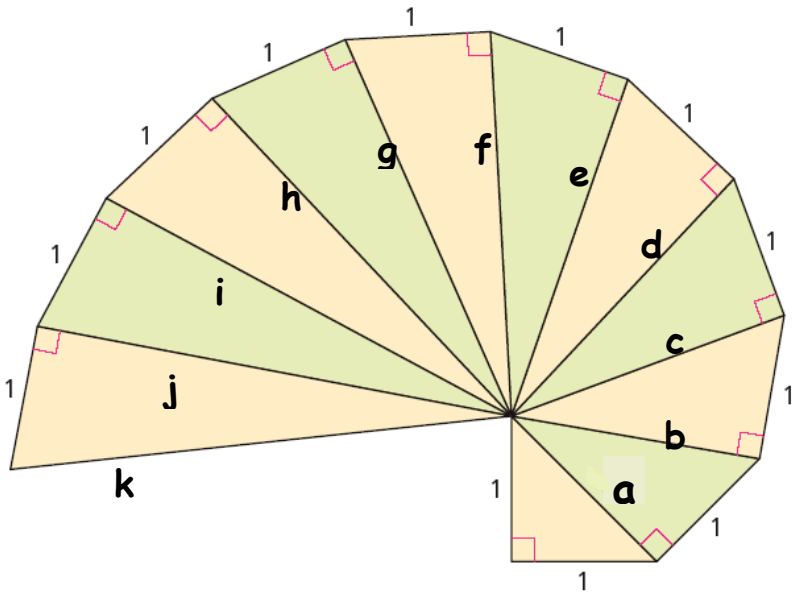
Homework: Using the Pythagorean Theorem

You know from your prior work that it is not possible to represent irrational numbers precisely as either terminating or repeating decimals. Decimal approximations for irrational numbers are infinite strings of numbers with no discernible patterns.

Surprisingly, however, it *is* possible to construct line segments that have lengths that are irrational numbers. In this homework assignment, you will use the method created by Theodorus, an Ancient Greek who was a follower of Pythagoras, to construct such segments.

Start with an isosceles right triangle with 1 cm legs. Find the length of the hypotenuse (**a**) and write it along the hypotenuse.

Now, build a new right triangle with **a** as the base and height 1 cm. (See diagram.) Find the length of the hypotenuse **b** of the new triangle and record it.



Continue in this manner, using the hypotenuse of the previous triangle as the base of a new triangle. (See diagram.)

1. Find the lengths of segments *c* through *k* and write them on the diagram.

2. What pattern do you notice in the lengths of the hypotenuses of the right triangles? What is the length of the hypotenuse of the 20^{th} triangle? The 100^{th} triangle? The nth triangle?

3. How could you use Theodorus's method to locate $\sqrt{2}$ on a number line? $\sqrt{3}$? $\sqrt{5}$? The square root of any number that is not a perfect square?

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